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An N=$\not 2$ SUSY Gauge Model for Dynamical Breaking of the Grand Unified SU(5) Symmetry

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Abstract

We construct an extension of the recently proposed dynamical model for the breaking of $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge symmetry, in which a pair of massless chiral supermultiplets for Higgs doublets are naturally obtained. We point out that a model at a specific point in the parameter space of superpotential is regarded as a low-energy effective theory of an N=$\not 2$ supersymmetric gauge model for the strongly interacting hypercolor sector.

Recently, a dynamical breaking scenario of the grand unified (GUT) gauge symmetry has been proposed based on a supersymmetric (SUSY) hypercolor $SU(3)_H$ gauge theory, where six flavors of quarks Q and antiquarks $\overline{Q}$ interact strongly at the GUT scale [1, 2]. The dynamical breaking of $SU(5)_{GUT}$ also produces a pair of massless composite color-triplet states simultaneously. A use of the missing partner mechanism [3] yields a pair of massless Higgs doublets, giving large masses to the color-triplet partners.

The model assumes, in addition to Q and $\overline{Q}$, a hypercolor-singlet chiral multiplet Σ in the adjoint representation of $SU(5)_{GUT}$ to get rid of unwanted Nambu-Goldstone multiplets. The Σ is easily extended to the $(\mathbf{35}+\mathbf{1})$ representation of the global SU(6) by introducing a pair of $\mathbf{5}$ and $\mathbf{5}^*$ and two singlets of $SU(5)_{GUT}$. Although this extended model is more complicated than the previous one, it seems to suggest a more fundamental theory at the short distance [4]. Namely, this extended model may be regarded as the dual theory of an original SU(3) gauge theory with six flavors of quarks q and $\overline{q}$, but without Σ [5].

In this letter, however, we propose a different view on the extended model. We show that a model with a specific choice of parameters in the superpotential is identical to the low-energy effective theory of an N=$\not{2}$ SUSY model for the strongly interacting gauge sector.

The previous model [1, 2] is based on a supersymmetric hypercolor $SU(3)_H$ gauge theory with six flavors of quarks Q^A_α in the $\mathbf{3}$ representation and antiquarks $\overline{Q}^\alpha_A$ in the $\mathbf{3}^*$ representation of the hypercolor $SU(3)_H$ (α=1-3 and A=1-6). The first fives of Q^A_α and $\overline{Q}^\alpha_A$ (A=1-5) transform as $\mathbf{5}^*$ and $\mathbf{5}$ of the $SU(5)_{GUT}$, respectively. In addition to the quarks and antiquarks we introduce a hypercolor singlet Σ^A_B $(A, B = 1-5)$ in the $\mathbf{24}$ representation of $SU(5)_{GUT}$ to avoid unwanted Nambu-Goldstone multiplets [1, 2].

It is a straightforward task to extend the Σ^A_B to the $\mathbf{35}+\mathbf{1}$ representation of SU(6) under which Q^A_α and $\overline{Q}^\alpha_A$ transform as $\mathbf{6}^*$ and $\mathbf{6}$, respectively. We introduce a new pair of $\mathbf{5}+\mathbf{5}^*$ and two singlets of $SU(5)_{GUT}$ and combine all of them with the adjoint $\mathbf{24}$ to form $\mathbf{35}+\mathbf{1}$ of the global SU(6).

We assume the following renormalizable superpotential;

$$W = \lambda Q^A_\alpha \Sigma^B_A \overline{Q}^\alpha_B + \mu \mathrm{Tr}\Sigma + m_1 \mathrm{Tr}\Sigma^2 + m_2 (\mathrm{Tr}\Sigma)^2 + \lambda_1 \mathrm{Tr}\Sigma^3$$

$$+\lambda_2 \mathrm{Tr}\Sigma^2 \mathrm{Tr}\Sigma + \lambda_3 (\mathrm{Tr}\Sigma)^3 \quad (A, B = 1-6), \tag{1}$$

so that this model has a global SU(6)×U(1) symmetry in the limit of the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge coupling g_5 vanishing ($g_5 = 0$). The U(1) should be gauged to have an unbroken $\mathrm{U}(1)_{\mathrm{Y}}$ gauge symmetry in the vacuum described bellow [1, 2].

This model contains seven complex parameters (λ, μ, m_1, m_2, λ_1, λ_2, and λ_3) in the superpotential. Two phases of λ and μ^2, for example, are absorbed to the fields Q_α^A and Σ_B^A. At the general point of the parameter space we always have the following vacuum,

$$\overline{Q}_A^\alpha = \begin{pmatrix} v & 0 & 0 & 0 & 0 & 0 \\ 0 & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & 0 & 0 \end{pmatrix}, \qquad Q_\alpha^A = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & v \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Sigma_B^A = \begin{pmatrix} 0 & & & & & \\ & 0 & & & 0 & \\ & & 0 & & & \\ & & & w & & \\ & 0 & & & w & \\ & & & & & w \end{pmatrix}. \tag{2}$$

In this vacuum our gauge group is broken down to the standard-model gauge group as

$$\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)_{\mathrm{H}} \times \mathrm{SU}(3)_{\mathrm{H}} \quad \rightarrow \quad \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}},$$

where the unbroken gauge group $\mathrm{SU}(3)_{\mathrm{C}}$ and $\mathrm{U}(1)_{\mathrm{Y}}$ are linear combinations of the $\mathrm{SU}(3)_{\mathrm{H}}$ and the SU(3) subgroup of $\mathrm{SU}(5)_{\mathrm{GUT}}$ and the $\mathrm{U}(1)_{\mathrm{H}}$ and the U(1) subgroup of $\mathrm{SU}(5)_{\mathrm{GUT}}$, respectively. The GUT unification of the gauge coupling constants in the standard model is practically achieved by taking sufficiently large gauge coupling constants of $\mathrm{SU}(3)_{\mathrm{H}}\times\mathrm{U}(1)_{\mathrm{H}}$ [1, 2].

In this vacuum (2) three pairs of the Q_α^A and $\overline{Q}_A^\alpha$ ($A = 4-6$) acquire masses λw. The integration of these massive quarks and antiquarks yields an $N_f = N_C = 3$ gauge theory where N_f and N_C are the numbers of flavor and color, respectively. As shown by Seiberg [6] the classical vacuum given in Eq. (2) is slightly modified by instanton effects in the case of $N_f = N_C$. However, the basic structure of the vacuum is not changed and

the standard-model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ remains unbroken even in the quantum vacuum.

Since the global SU(6) (in the limit g_5=0) is broken down to $SU(3)_C \times SU(3)$ in this vacuum, we have three true and two pseudo Nambu-Goldstone multiplets. The two true Nambu-Goldstone multiplets transforming as $(\mathbf{3},\mathbf{2})$ and $(\mathbf{3}^*,\mathbf{2})$ under $SU(3)_C \times SU(2)_L$ are absorbed to the $SU(5)_{GUT}$ gauge multiplets and the last one to the broken U(1) gauge multiplet. Thus, there remains a pair of pseudo Nambu-Goldstone multiplets which transforms as $\mathbf{3}$ and $\mathbf{3}^*$ of $SU(3)_C$. As shown in the previous paper [1, 2], these $\mathbf{3}$ and $\mathbf{3}^*$ acquire GUT-scale masses together with the $\mathbf{3}$ and $\mathbf{3}^*$ in the standard Higgs multiplets $H_A(\mathbf{5})$ and $\overline{H}^A(\mathbf{5}^*)$ of $SU(5)_{GUT}$. Thus, there is a pair of Higgs doublets left in the massless spectrum. It is easily proved by using the SUSY nonrenormalization theorem [7] that this pair of Higgs doublets is exactly massless even in the quantum vacuum as far as the SUSY is unbroken.

Now, we show our main point in this letter. There is a peculiar hypersurface in the parameter space of the extended model, on which $U(1)_R$ symmetry is restored. This surface is defined by $\mu = \lambda_1 = \lambda_2 = \lambda_3 = 0$ and hence it consists of only 3 parameters λ, m_1, and m_2 in Eq. (1). Under the $U(1)_R$, each fields, Q^A_α, $\overline{Q}^\alpha_A$, and Σ^A_B transform as

$$\begin{aligned} Q^A_\alpha(\theta) &\rightarrow \mathrm{e}^{-\frac{i}{2}\alpha} Q^A_\alpha(\theta \mathrm{e}^{i\alpha}), \\ \overline{Q}^\alpha_A(\theta) &\rightarrow \mathrm{e}^{-\frac{i}{2}\alpha} \overline{Q}^\alpha_A(\theta \mathrm{e}^{i\alpha}), \\ \Sigma^A_B(\theta) &\rightarrow \mathrm{e}^{-i\alpha} \Sigma^A_B(\theta \mathrm{e}^{i\alpha}). \end{aligned} \tag{3}$$

On this hypersurface we have the vacuum defined in Eq. (2) only when $m_2 = -\frac{1}{3}m_1$ is satisfied. Thus a fine tuning of the parameters is required to obtain the desired vacuum given in Eq. (2). However, we show that such a specific point in the parameter space corresponds to a low-energy effective theory of an N=$\not{2}$ SUSY gauge model for the strongly interacting sector.

The N=$\not{2}$ SUSY model discussed by Leigh and Strassler [8] contains an N=2 gauge multiplet of $SU(3)_H$ and six (=$2N_C$) hypermultiplets $(Q^A_\alpha, \overline{Q}^\alpha_A)$. The N=2 gauge multiplet consists of an $SU(3)_H$ adjoint chiral superfield X^α_β and the $SU(3)_H$ gauge multiplet in the N=1 SUSY theory. Then, the superpotential is written in terms of the N=1 SUSY fields

as

$$W = gQ^A_\alpha X^\alpha_\beta \overline{Q}^\beta_A, \tag{4}$$

where g is the $\mathrm{SU}(3)_\mathrm{H}$ gauge coupling constant. The N=$\not{2}$ SUSY model is defined by introducing the mass term for X^α_β [4],

$$\delta W = \frac{m_X}{2}\mathrm{Tr}X^2, \tag{5}$$

which breaks the N=2 SUSY down to the N=1.

The integration of X^α_β leads to an effective superpotential

$$\begin{aligned} W' &= -\frac{g^2}{m_X}\left(Q^A_\alpha \overline{Q}^\beta_A\right)\mathrm{T}^{i\alpha}_\beta \mathrm{T}^{i\gamma}_\delta \left(Q^B_\gamma \overline{Q}^\delta_B\right) \\ &= \frac{g^2}{2m_X}\left(\left(Q^A_\alpha \overline{Q}^\alpha_B\right)\left(Q^B_\beta \overline{Q}^\beta_A\right) - \frac{1}{3}\left(Q^A_\alpha \overline{Q}^\alpha_A\right)\left(Q^B_\beta \overline{Q}^\beta_B\right)\right), \end{aligned} \tag{6}$$

where T^i is a generator matrix of the $\mathrm{SU}(3)_\mathrm{H}$ Lie algebra. This superpotential is rewritten by using an auxiliary field Σ^A_B as

$$W' = \lambda Q^A_\alpha \Sigma^B_A \overline{Q}^\alpha_B + \frac{\lambda^2 m_X}{2g^2}\left[\mathrm{Tr}\Sigma^2 - \frac{1}{3}(\mathrm{Tr}\Sigma)^2\right]. \tag{7}$$

This corresponds exactly to the superpotential at the specific point in the parameter space of our extended model on which the desired vacuum given in Eq. (2) is obtained. In fact, we drive F-term flatness conditions for vacua directly from the superpotential (4) and (5) as,

$$X^\beta_\alpha \overline{Q}^\alpha_A = Q^A_\alpha X^\alpha_\beta = 0, \tag{8}$$

$$m_X X^\beta_\alpha = -g(\overline{Q}^\beta_A Q^A_\alpha - \frac{1}{3}\delta^\beta_\alpha \mathrm{Tr}\overline{Q}Q), \tag{9}$$

which yield the solution for Q^A_α and $\overline{Q}^\alpha_A$ given in Eq. (2) together with D-term flatness conditions.

At the short distance the Σ^A_B is an auxiliary field of dimension two in the N=$\not{2}$ SUSY model while the Σ^A_B in the extended model is a propagating canonical field of dimension one. However, it is very much plausible that the radiative corrections generate the kinetic term for the auxiliary field Σ^A_B at the considerably long distance like at the GUT scale.

Therefore, we believe that both theories become indistinguishable at the low energies as point out in Ref. [7][1]

Since the N=$\not{2}$ SUSY model does not need a fine-tuning of parameter to have the desired vacuum, we consider that the extended model at a specific point in the parameter space is a low-energy effective theory of the more fundamental N=$\not{2}$ SUSY model.

We should note that there is a massless Nambu-Goldstone multiplet corresponding to the $U(1)_R$ breaking. As a consequence there is a exactly flat direction keeping the vacuum-expectation value v in Eq. (2) undetermined unless the explicit breaking term of the $U(1)_R$ is introduced.

Finally we comment on the nucleon's instability. Unlike the previous model [1, 2] we do not have a suppression mechanism for the dangerous d=5 operators of the nucleon decay in the present model. Therefore, we hope that the coming nucleon-decay experiments will select one of these two different models.

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[1] Precisely speaking, however, dynamics in our model is not the same as that in the N=$\not{2}$ SUSY model in Ref. [7] because of the presence of $U(1)_H$ gauge interaction, even when one turns off the weakly interacting $SU(5)_{GUT}$. Nevertheless, it is amusing to note that the gauge coupling constant g_{1H} of the $U(1)_H$ is vanishing at the very long distance and hence the argument in Ref. [7] may be applicable even in our N=$\not{2}$ SUSY model if one could take such a long distance.

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